CHILDREN'S STRATEGIES AND REASONING PROCESSES IN SOLVING NOVEL COMBINATORIAL AND DEDUCTIVE PROBLEMS

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This paper reports on a current study investigating 9 to 12 year-olds' strategies and reasoning processes in solving novel combinatorial and deductive problems. Equal numbers of children were selected from low, average, and high achievers in school mathematics and were individually administered five sets of problems, namely, two sets of combinatorial problems (2-dimensional and 3-dimensional) and three sets of deductive reasoning problems. Both problem types were presented in "hands-on" and written formats. The written problems were isomorphic to the hands-on examples. The nature of children's strategies and reasoning processes in solving these problems is addressed. Of particular interest are the differences in the responses of children classified as low and high achievers in school mathematics.

Research on children's mathematical problem solving has focussed largely on their ability to solve routine problems which call for a standard method of solution and which offer little opportunity for observing the processes of learning (e.g., Baranes, Perry, & Stigler, 1989; Riley & Greeno, 1988). While the mathematics education community has espoused the importance of problem solving during the past two decades (e.g., National Council of Teachers of Mathematics, 1989), comparatively little research has addressed novel mathematical problem solving where children do not have an efficient solution procedure and must develop their own strategies for goal attainment. Investigations into children's reasoning processes in solving novel problems can provide valuable data on their potential for mathematical learning -- information which might otherwise go undetected in routine mathematical activities.

Current theories on children's learning and development strongly support the call for novel problem solving in the mathematics curriculum. Several studies have portrayed children as self-directed learners whose problem-solving efforts are likened to those of a scientist, creating theories-in-action which they challenge, modify, and extend on their own (Brown & Palinscar, 1989; Burton, 1992; Carey, 1985; Gelman & Brown, 1986; Karmiloff-Smith, 1984). Studies of very young learners show that they possess some powerful problem-solving skills. For example, they are able to direct their attention to tasks that interest them, they independently monitor their progress towards goal attainment and modify their actions accordingly, and they persist at a problem while moving through increasingly sophisticated levels of solution procedures (Andreassen, Kelly, & Waters, 1991; Brown & Reeve, 1987; Deloache, Sugarman, & Brown, 1985; Gelman & Greeno, 1989; Klahr, 1985). If young children display these skills, it could be assumed that older children will demonstrate similar abilities in novel situations provided they are motivated to do so.

Previous studies (e.g., English,1988, 1991a, 1993) had shown that primary school children can generate their own strategies for solving novel, two-dimensional (X x Y) and three-dimensional (X x Y x Z) combinatorial problems set within a meaningful context (dressing toy bears in different combinations of clothing items and tennis rackets). The present study further explored this ability by extending the hands-on problems to include written isomorphic examples. Of interest here was whether children could recognise the similarity in problem structures and could apply analogical reasoning (Halford, 1992) in solving the isomorphic examples.

Novel problems involving deductive reasoning were also included in this study, given that previous work (e.g. English, in press) had highlighted children's ability to reason deductively in solving logical and illogical syllogisms. Furthermore, since the combinatorial and the deductive domains both represent novel situations for children and thus demand considerable use of general reasoning processes, it was considered worthwhile to compare children's performance across the two areas.

In addition to their novelty feature, combinatorics and deduction were chosen for their mathematical and developmental significance. The combinatorial domain, involving the selection and arrangement of objects in a finite set, comprises a rich structure of significant mathematical principles which underlie several areas of the mathematics curriculum, including counting, computation, and probability. The domain has also featured prominently in theories of cognitive development. The establishment of a combinatorial system plays a significant role in Piaget's theory of cognitive growth, heralding the onset of formal thought (e.g., Inhelder & Piaget, 1958; Piaget, 1957; Piaget & Inhelder, 1975).

Deductive reasoning problems were included because of their importance in the development of mathematical thinking. There is the assumption that because mathematics is a logically developed discipline, the logic will be absorbed by the students as they pursue a study of the formal content areas (Burton, 1984). However this is rarely the case. A facility to think mathematically is not a natural consequence of acquiring mathematical content. This facility relies on the development of a number of mathematical processes or operations which are applied to a range of content areas (NCTM, 1989). The ability to detect relationships, make hypotheses and generalisations, and test conjectures are major components of mathematical thinking and problem solving.

METHOD

Subjects

Three hundred and thirty-six children in grades 4 through 7 have been involved in the study. Four age groups are represented here (84 children per group): 9 yrs 0 mths to 9 yrs 6 mths, 10 yrs 0 mths to 10 yrs 6 mths, 11 yrs 0 mths to 11 yrs 6 mths, 12 yrs 0 mths to 12 yrs 6 mths. The children were selected from four state schools and three non-state schools in low to middle socio-economic suburbs of Brisbane. Equal numbers of children were selected from the low, average, and high achievement levels in mathematics (as defined by the class teacher).

Instruments

Five sets of problems (three problems per set) were developed for the study, as follows:
Set 1: one 2-D (X x Y) and two 3-D (X x Y x Z) combinatorial problems (hands-on)
Set 2: three written combinatorial problems, isomorphic to the first set
Set 3: three hands-on deductive reasoning problems (non-numerical)
Set 4: three written deductive reasoning problems, isomorphic to Set 3 (non-numerical)
Set 5: three written, numerical deductive reasoning problems.

Set 1 required children to dress toy bears in all possible combinations of coloured tops and pants (2-D examples) or coloured tops, pants, and tennis rackets (3-D examples). The bears were made of thin wood and were placed on a stand so that the children could clearly see their completed combinations. The number of possible combinations ranged from 6 to 12. The nature of the written combinatorial examples (Set 2) is illustrated in Figure 1.

The Select-A-Card company plans to make new boxes of greeting cards.
In each box there will be greeting cards that are;
either GREEN or YELLOW, and have
either CHRISTMAS greetings or BIRTHDAY greetings or EASTER greetings, and have either GOLD LETTERING or SILVER LETTERING.
How many different greeting cards will there be in each box?

Figure 1. Example of a written three-dimensional combinatorial problem

Sets 3 and 4 required children to work through a series of clues to solve problems of arrangement or association. For example, in one of the hands-on problems, children were provided with regular playing cards and given a series of clues as to how the cards were to be arranged. An example of a written deductive problem involving associations appears in Figure 2.

Four famous sports people entered a television studio. One was a tennis player, one was a swimmer, one was a golfer, and the other was a chess player. Use the clues to find out who played what sport.

* *Mr Bowler is not good at chess.*
* *Both Mr Big's and Ms Ace's sports involve a ball.*
* *Ms Fish can't swim at all.*
* *Neither Ms Ace nor Ms Fish play tennis.*

Figure 2. Example of a written deductive reasoning problem (non-numerical)

The final set of problems was designed to assess whether the inclusion of number interferes with children's deductive reasoning processes. A sample problem from this set appears in Figure 3.

Sue bought two items from the tuckshop. Use the clues to find out which two items she bought.

icecream.........85c	*bun........45c*	*drink........92c*
chocolate..........50c	*sandwich...90c*	*apple.......20c*

CLUES:

** Sue only had $1.50 to spend. * She does not like sandwiches.*
** One item cost less than 50c. * If Sue buys an icecream, she always buys a drink as well. * If Sue buys a bun, she always buys a chocolate as well.*
** After buying her two items,Sue had more than 50 cents change.*

Figure 3. Example of a written numerical deductive reasoning problem

Procedure

The children were presented the problems on a one-to-one basis in two or three sessions on consecutive days. The order in which the children were presented the problems was counterbalanced for Sets 1 to 4. The children were expected to complete the problems without assistance from the research assistant. To avoid any reading difficulties for the child, the research assistant read aloud each of the written problems which were presented on an activity sheet. After the two sets of combinatorial problems had been completed, the research assistant asked the child if solving one set of problems helped him/her to solve the other set. The child was also asked to identify ways in which the two sets of problems were similar. This procedure was repeated with the deductive reasoning problems, sets 3 and 4. The responses of all children were videotaped for subsequent analysis.

Data analysis

The children's responses on the combinatorial problems were analysed in terms of their strategies, their written procedures, goal attainment, and their detection of similarities between problem sets (structural or surface level).

The children's responses on the deductive reasoning problems were analysed in terms of their:
1. overall plan of attack, such as drawing an appropriate diagram;
2. specific problem-solving and reasoning processes, including, seeing connections and relationships among items of information, use of means-ends analysis, consideration of alternatives; 3. application of mathematical knowledge and skills; 4. use of appropriate repairs;
5. checking and monitoring processes;
6. detection of similarities between problem sets.

Results

As this study is still in progress and the data analysis incomplete, only a selection of findings can be reported here. Children's strategies for solving the combinatorial problems had been identified previously (e.g. English, 1992; 1993) and were confirmed in the present study; they are revisited briefly here.

Children's combinatorial strategies

The interesting feature of the strategies children used in solving the combinatorial problems was their diverse nature, reflecting varying levels of domain knowledge. Their strategies for solving the two-dimensional problems ranged from inefficient, trial-and-error procedures, to a uniform cyclic pattern in item selection (e.g. red top, blue top, green top, red top, blue top, green top.....), and finally, to a sophisticated odometer strategy which is the most efficient for generating all possible combinations. Here, one item is held "constant" while the other is systematically varied (e.g. red top/blue pants, red top/green pants, red top/yellow pants.....). This process is repeated until all constant items have been used. Children's three-dimensional strategies also reflected a hierarchy of five, increasingly complex, procedures with the most advanced involving the simultaneous use of two constant items. A detailed analysis of these strategies can be found in English (1993).

At this stage of the analysis, it appears that children classified as low achievers in mathematics gained from completing the hands-on combinatorial problems (set 1) prior to the written examples (set 2). The order of problem presentation did not appear to affect the responses of the average and high achievers. Children from each achievement level were able to transfer their combinatorial strategies from one problem set to another, irrespective of the order in which these were presented. When solving the written combinatorial problems, children mainly employed a listing procedure. Some children however, wrote each of the possible combinations in set form, with the names of the items encircled. The use of a tree diagram or array format was rare, indicating that such procedures need to be specifically taught. When asked if they could see any similarities between the hands-on and written combinatorial problems, many children were unable to recognise the common underlying structural similarities. They tended to focus on the surface features, this being a frequently observed feature of novice problem solving (Biggs & Moore, 1993).

Children's deductive reasoning processes

A particularly interesting finding to date pertains to the deductive reasoning processes of children classified as low achievers in mathematics. For example, these lower achievers appeared to quickly see relationships and connections between items of information, using these to streamline the solution process. The self-monitoring processes of many of the lower achievers also seemed comparable, if not better, than those of the other children. The response of 10 year-old Gabrielle below, classified as a low achiever in mathematics, serves as an example here. She solved the problem of Figure 2 as follows:

Gabrielle listened to the clues given and listed down the names of the four people. Her first step was to write the word, "golf" beside the name, "Ms Ace." Gabrielle explained that she did this, "Because Ms Ace doesn't like tennis and she uses something with a ball." Gabrielle then claimed that Ms Fish plays chess, "because she can't play golf and she can't swim and she can't play tennis." This was followed by the matching of Mr Big with tennis ("because he uses a ball") and Mr Bowler with swimming.

It was interesting to observe the responses of children like Gabrielle and compare them with those of her high achieving counterparts. Several of the latter appeared quite inefficient in solving these deductive problems (in some cases, transcripts of their responses on just one example took several pages). They appeared unable to work logically through the clues, failed to make connections, moved repeatedly from one clue to another, and made numerous repairs during their solution attempts.

The order in which children were presented the deductive reasoning problems had a bearing on their performance. Irrespective of their achievement level, children who were administered the hands-on deductive problems prior to the corresponding written examples appeared more competent in solving the written examples than those children who received the problems in the reverse order.

Children's procedures in solving the written examples included the use of diagrams and lists to display the arrangements or associations of items. Some children worked the written examples mentally and recorded only their answer. This suggests that they were using quite sophisticated reasoning processes, given the nature and amount of information to be processed. With the exception of one child, the children did not use a matrix to solve the problems. Given the obvious benefits of such representational formats (Novick, 1990; Polich & Schwartz, 1974), it is important that children be exposed to these after prior experiences with more informal methods.

When recognising similarities between the problem sets, children in each of the achievement levels performed better here than on the combinatorial problems. They appeared better able to detect the underlying structural similarities between the deductive problems. The final set of deductive problems which draws upon children's number sense, appeared more challenging even though these problems have a similar structure to the previous two sets. Weaknesses in children's knowledge of numeration and understanding of terminology such as, "3 times as old as", hampered their progress here.

CONCLUDING POINTS

Given that this study is still in progress, it is difficult to draw major conclusions. Nevertheless, the study does highlight children's ability to reason logically in solving novel problems and, in the process, independently develop more sophisticated procedures. Of significance is the mathematical potential of children classified as low achievers in school mathematics, this potential being particularly evident in their responses on the deductive reasoning problems. Their performance here highlights the importance of broadening the mathematics curriculum to include a range of novel problems; such problems should provide all children with the opportunity to display their reasoning skills, generate their own solution procedures, discover new ideas and principles, and in essence, to take control of their learning (English, 1991b).

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